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Enclosure 1

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Annual Report for 2000, Army Research Office
Stochastic Control Problems in Mobile Communications

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1. Numerical Methods of Stochastic Control. The book [6] contains the most comprehensive development of numerical algorithms and associated convergence proofs for a large part of the current forms of stochastic control problems, and has been quite popular. A second edition was completed. In addition to the broad coverage of the previous edition, it gives numerical algorithms and proofs for problems where the variance term is controlled, and for jump–diffusions where the jump is controlled. The latter problem has arisen in recent applications to communications theory. The standard use of the Poisson measure driven model is no longer adequate, and a general theory is developed. Additionally, the book contains a thorough development for deterministic problems which arise in control and the calculus of variations, and includes discontinuous or unbounded dynamical terms, with applications to image reconstruction, large deviations, and elsewhere. The algorithms are about the fastest and most stable available, and there are convergence proofs for all of them.

2. Routing and service control for large dimensional networks. Key issues in our investigations include: (i) the development of approximate models that allow for explicit (or nearly explicit) construction of the optimal routing/service policies, and (ii) robustness and the ability to deal with model perturbations. The problems considered encompass a number of different formulations, each of which emphasizes different qualitative properties of the resulting controlled network. These include risk-sensitive control and the control of rare events, optimal control of "fluid" models, and optimally robust control of such models. "Fluid" models are approximate models that are obtained under a law of large numbers scaling. In all cases the system model is constrained, and in most problems this is a dominant feature. To handle the constraints, we model the state dynamics of the approximate (or limit) models in terms of an appropriate Skorokhod Problem and corresponding constrained ordinary or stochastic differential equations. The different problem setups (e.g., control of rare events versus control of fluid models) lead to problems of the same basic form, which is a Skorokhod Problems with (relatively) simple dynamics and simple cost structures.

Although the general theory of optimal control for such network approximations is still in the early stages of development, there are already a number of interesting and important problem classes for which explicit solutions have been found. These classes of problems provide analogues in the setting of constrained processes for the familiar linear/quadratic problems from the theory of unconstrained processes. As such, they may eventually play as important a role in the developing theory for problems with constraints. Examples

include references [4, 5] The collection of problems with explicit solutions will continue to expand, as we become more familiar with the geometries and cost structures that are natural for constrained processes (from the point of view of explicitly solvable problems), and the new techniques which allow for their solution. Reference [4] deals with a controlled constrained ordinary differential equation, and with a cost that depends only on the control. Such problems occur in the (ordinary) control of fluid models, such as control of a network with the objective of reducing backlogs in minimum time. Our analysis gives an explicit finite-dimensional representation of the value function, and identifies all optimal controls. Reference [5] considers the problem of characterizing the manner in which rare events occur in networks under heavy traffic. It shows how time reversal arguments can be applied to rewrite the control problem defined by a straightforward large deviations analysis into the form of [2], and then shows how explicit finite-dimensional solutions can be obtained for some interesting classes of problems (in arbitrary dimension). A three dimensional example is worked out in detail for illustrative purposes. Reference [2] deals with related problems of stability of constrained processes.

3. Polling, and applications to wireless, controlled jumps, and stability. Consider a queueing system with many queues, and one server which allocates its time among the queues to minimize some cost. Usually, it is assumed that the queues are always available, and the server can allocate at will. In modern applications to wireless or military systems, there are randomly occurring periods where a source is unavailable, and this must be accounted for in the polling policy. The possibility of such "vacations" complicates the problem enormously, and there is little literature on the subject, despite the vast amount on polling. Again, due to the complexity of the basic problem, we analyze [1] it in the heavy traffic regime where the server has little idle time over the average requirements. The scaled total workload process converges to a controlled limit diffusion process with jumps, which are due to the effects of the vacations. The control enters the dynamics only via its value just before a vacation begins; hence it is only via the the random jump value that the control affects the dynamics. This type of model has not received any attention. The limit problem can be represented in terms of a one dimensional reflected jump–diffusion, no matter the number of queues, which greatly simplifies the limit and enhances the value of the approach. Many results of considerably broader use were obtained, such as averaging principles which assure the validity of a fluid approximation during the vacation. Explicit analytical and numerical solutions were obtained in some simple but important cases, and they are not intuitive, owing to the anticipation of the effects of the vacations. Stability and unreliable channels were treated, under general conditions on the data.

4. Heavy traffic modeling for communications systems. We have been writing a comprehensive tratise on the use of heavy traffic methods for the simplification and analysis of complex communications systems and related queueing type networks. The aim of the book is the development of the heavy traffic approach to the modeling and analysis of queueing networks, both controlled and uncontrolled, and many applications to computer, communications, and manufacturing systems. The methods exploit the multiscale

structure of the physical problem to get approximating models that have the form of reflected diffusion processes, either controlled or uncontrolled. These approximating models have the basic structure of the original problem, but are significantly simpler. Much of inessential detail is eliminated (or "averaged out"). They greatly simplify analysis, design, and optimization and yield good approximations to problems that would otherwise be intractable, under broad conditions.

One is concerned with rather general interarrival and service intervals, complicated network topologies and routing schemes, control or optimal control mechanisms, effective numerical methods, stability questions, state dependence, service interruptions, non-Markov models, and other complications such a those due to finite or shared buffers, feedback, blocking, or the balancing of the demands or the scheduling of several different job classes. Correlations in the service times for the same job as it proceeds from processor to processor and other forms of correlation are also current issues, as are randomly varying parameters, perhaps due to varying environmental factors. The demands of modern high-speed and high-performance communications pose very serious challenges, since they must operate with very small error rates. Although there is considerable effort devoted to these problems, a great many of the most significant remain intractable. When one wishes to apply optimal control methods, the exact physical model is often much too complicated for an exact solution to be obtained.

Although one would like to solve the problems without any type of approximation, asymptotic methods are often the best solution (or even the only hope). We are concerned with the approximation of controlled or uncontrolled queueing networks by reflected diffusion or jump–diffusion processes. Diffusion approximations have long been an effective tool for simplifying or approximating physical or biological problems The reflected diffusion approximation simplifies the queueing network problem in many ways. It is Markovian and lower (often much lower) dimensional, numerical methods are available, and the structure is exhibited more clearly. Even formal analyses of complex problems that are guided by the insights and approximations of heavy traffic analysis can lead to impressive results.

5. Analysis and Control of Mobile Communications with Time Varying Channels in Heavy Traffic. Consider a system with a fixed number (K) of remote units and a single base transmitter with time varying (and perhaps correlated) connecting channels. Data to be transmitted to the remote units arrives according to some random process and is queued according to its destination. The uplink is treated, for specificity. Power is to be allocated to the K channels in a time varying way. The modeling and control problem can be quite difficult. The channel time variations (fading) are fast and the bandwidth and data arrival rate are high. Owing to the complexity of the physical problem and the high speed of both the fading and arrival and service rates, an asymptotic or averaging method is promising. A heavy traffic analysis was done in [3].

By heavy traffic, we mean that on the average there is little server idle time and little spare power. Heavy traffic analysis has been very helpful in simplifying analysis of both controlled and uncontrolled problems in queueing and communications networks. It tends to eliminate unessential detail and focus on the fundamental issues of scaling and para-

metric dependencies. A variety of models are considered. The basic model assumes that the channel state is known or can be well estimated and that given the channel state there is a well defined rate of transmission per unit power. Then convergence of the controlled scaled queue lengths is shown. The scaling is different from the usual in heavy traffic work, and the limit Wiener process depends only on the channel state process and not on the data arrival process. The appropriate orders of reserve power and buffer size is given as well as suggested policies. The limit or approximating process is a controlled reflected diffusion which is much simpler than the original problem and facilitates understanding parametric dependencies, solutions and stability. There are new issues in computing the reflection direction. Additionally, a more general model is considered, where knowledge of the channel can be subject to random errors, delays and other perturbations as well as power scheduling constraints. The methods of proof and limit forms (hence the limit or approximating control problems) are similar.

The analysis and results do not depend on the details of coding, modulation or detection: These are represented by the transmission rate processes. This procedure allows considerable generality as well as focusing on what is essential.

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